# SIGNALING RECONSIDERED

by

Cai Hongbin, Ye Lixin

and

Jong Wai Lei

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3rd draft

# OUTLINE

- History
- Formal Model
- Nash Equilibrium
- Refinements
- Problems (i) wrong answer (ii) not robust (iii) how credible?
- Solution – the Local Credibility Test
- Application to a two types example
- Application as the number of types grows large
- Continuum of types
- Alternative approach – the strong LCT

# HISTORY

“Walrasian” equilibrium (Spence, Riley)

Screening equilibrium (Rothschild-Stiglitz)

formal game in which uninformed move first

Signaling equilibrium ( Cho-Kreps, Banks-Sobel, Kohlberg-Mertens Grossman-Perry, Mailath-Okuno-Fujiwara-Postlewaite)

formal game in which the informed player moves first

# MODEL

A signaler has a characteristic vector $(s_i, v_j)$

$s_i \in S = \{s_1, \ldots, s_n\}$, $v_j \in V = \{v_1, \ldots, v_m\}$.

# MODEL

A signaler has two characteristics $s_i \in S = \{s_1, \ldots, s_n\}$, $v_j \in V = \{v_1, \ldots, v_m\}$.

$s_i$ is her cost characteristic $v_j$ is her value characteristic (value to responders)

Responders know only the probability distribution $\pi_{ij} = \pi(s_i, v_j)$. This is common knowledge. **The two characteristics are negatively affiliated**.

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The higher the cost characteristic, the lower the marginal cost of action *y*. (single crossing)

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The higher the cost characteristic, the lower the marginal cost of action *y*. (single crossing)

The value of the transaction to the other agents is a linear function of the value characteristic $v$.

Thus the equilibrium response $r$ is a function of $\hat{v}$, the expected value of *v*, that is $r = r(\hat{v})$.

The higher is $\hat{v}$ the greater the response $(r'(\hat{v}) > 0)$

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$s_i \in S = \{s_1, \ldots, s_n\}$, $v_j \in V = \{v_1, \ldots, v_m\}$.

The equilibrium response $r(\hat{v})$ is an increasing function of the expected value of $v$.

Signaler's payoff is $U(s, r(\hat{v}), y)$

Assumptions:

$$U_2 > 0,\ U_3 < 0,\ \frac{\partial}{\partial s}(-\frac{U_3}{U_2}) < 0$$

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Signaler's strategy is a mapping $y(s)$

Seek a Nash Equilibrium in pure strategies. Suppose all *s* in $\hat{S}$ choose $\hat{y}$

Define $v(s) = \mathrm{E}\{v \mid s\}$ then responders observe $\hat{y}$, infer *s* is in $\hat{S}$ and hence infer that the expected value parameter is $\hat{v} = \mathrm{E}\{v(s) \mid \text{s in } \hat{S}\}$.

# SPENCE CONSULTANT MODEL

A consultant with characteristic vector $(s_i, v_j)$ has a signaling cost $c(s_i, y_j)$ and a marginal product $v_j$. Suppose that, in equilibrium, those with a signaling cost in $\hat{S}$ choose $\hat{y}$. Define the seller's "type" $t = v(s)$. The consultant's expected marginal product (her expected type) is then

$$\hat{t} = E\{t \mid t \in \hat{T}\} = E\{v(s) \mid s \in \hat{S}\}$$

Equilibrium response of firms is to pay her expected marginal product $\hat{t}$.
Signaler's payoff is therefore

$$U(t, \hat{t}, y) = \hat{t} - c(s, y) \equiv \hat{t} - C(t, y), \text{ where } t = v^{-1}(s)$$

Single crossing property: $MC(t_1, y) > MC(t_2, y)$

# NASH EQUILIBRIUM

## Proposition: Monotonicity -- $y(s)$ is non-decreasing

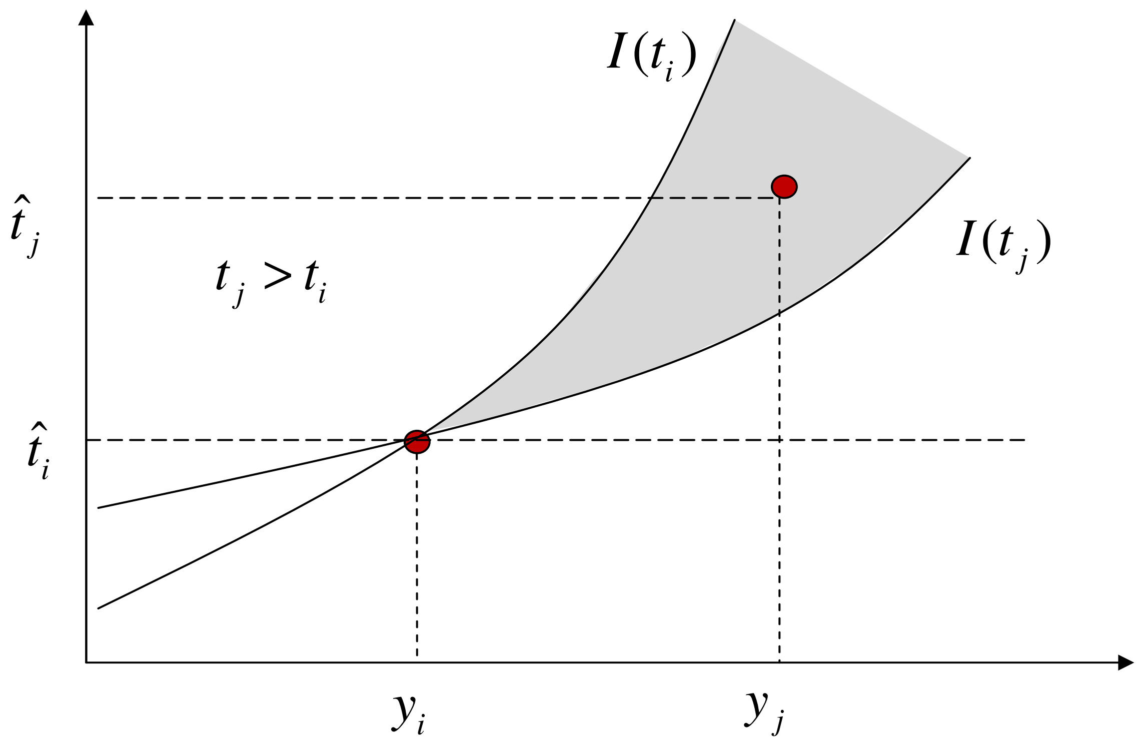


Fig. 4.1: Single Crossing and monotonicity

# NASH EQUILIBRIUM

## 2×2 example

| $\pi(s_i, v_j)$ | $v_1$ | $v_2$ | $\pi_i$ | $t_i = E\{v \mid s_i\}$ |
|---|---|---|---|---|
| $s_1$ | $\pi_{11}$ | $\pi_{12}$ | $\pi_1 = \pi_{11} + \pi_{12}$ | $t_1 = (\pi_{11}v_1 + \pi_{12}v_2) / \pi_1$ |
| $s_2$ | $\pi_{21}$ | $\pi_{22}$ | $\pi_2 = \pi_{21} + \pi_{22}$ | $t_2 = (\pi_{21}v_1 + \pi_{22}v_2) / \pi_2$ |

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Let $y(t),\ t \in T$ be an increasing mapping from type to signal. Define $\hat{t}_j = E\{t \mid y(t) = y(t_j)\}$. This is the expected marginal product of a type choosing $y(t_j)$. The mapping is a NE mapping if it is IC, that is, for all $t_i$ and $t_j \neq t_i$,

$$u(t_i, \hat{t}_i, y(t_i)) \geq u(t_i, \hat{t}_j, y(t_j))$$

# NASH EQUILIBRIUM - - Separating equilibrium (**continuum**)

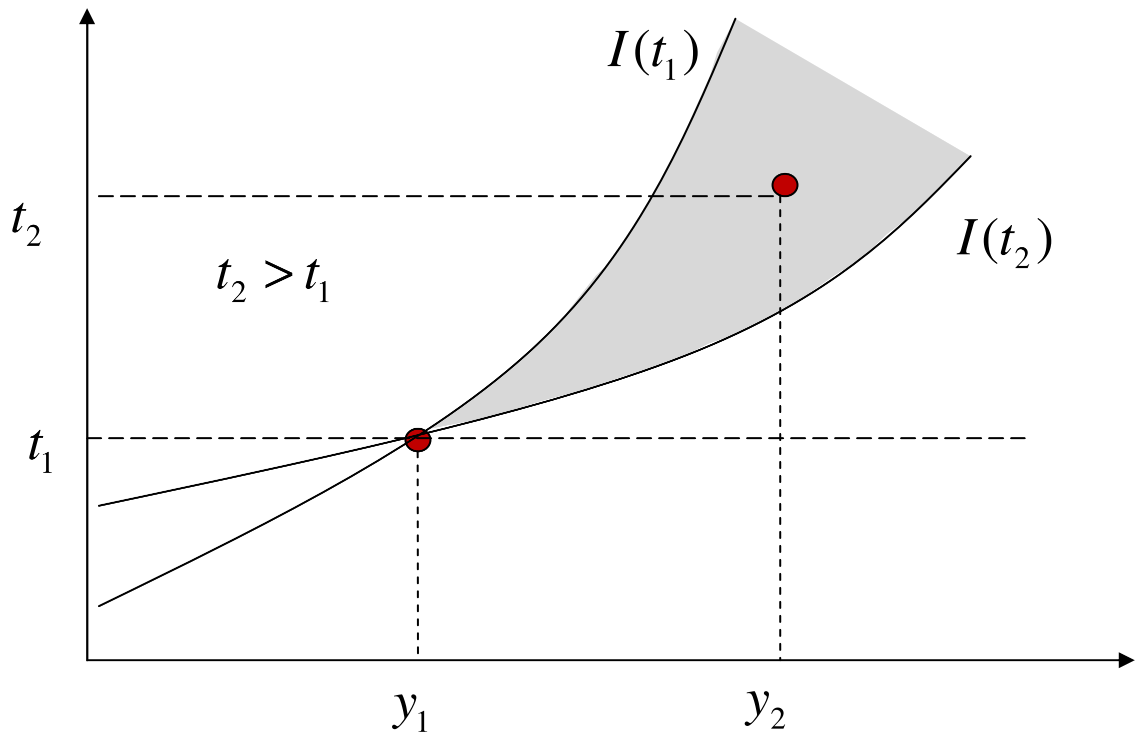
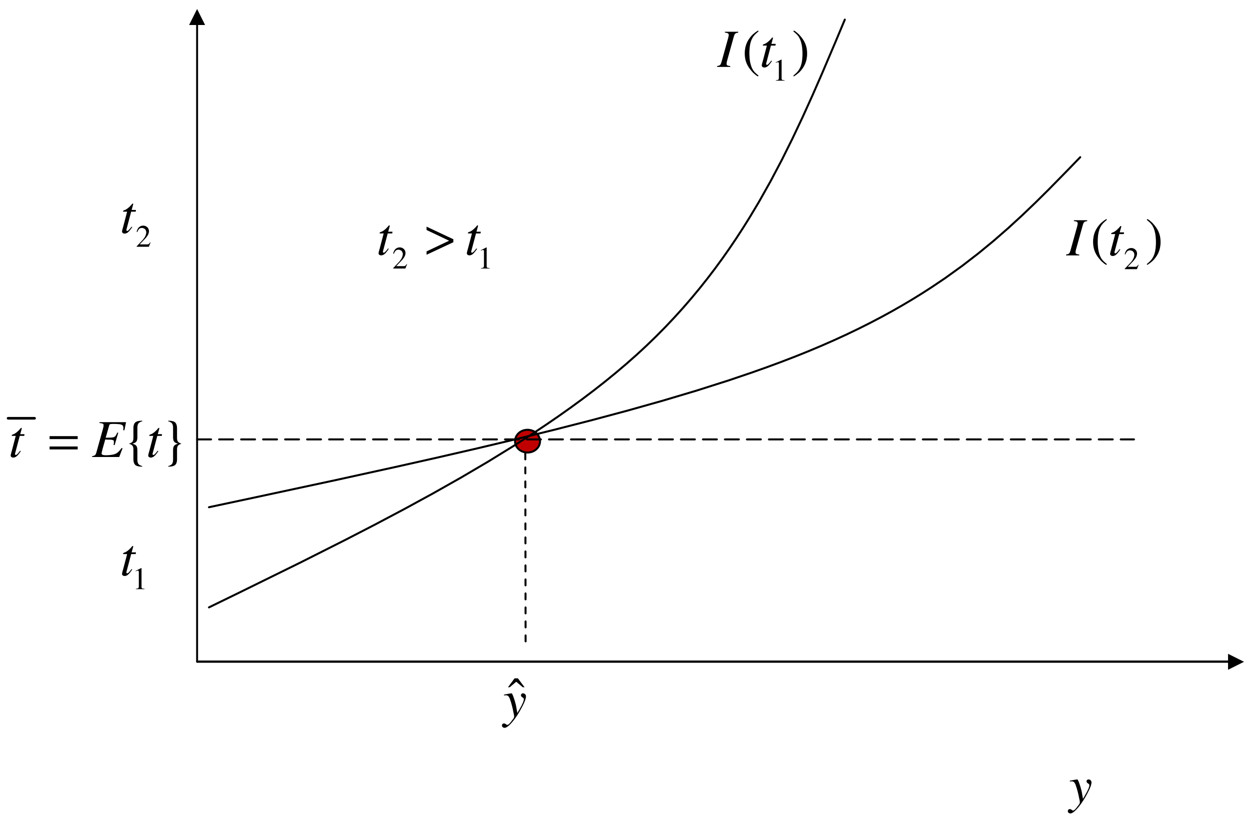



Fig. 4.1b: Separating Equilibrium

# NASH EQUILIBRIUM  - - Pooling Equilibrium (**continuum**)



# REFINEMENTS

## Intuitive Criterion (Cho-Kreps) "Equilibrium dominance"

The responders observe the out-of-equilibrium signal $\hat{y}$.

They ask whether there is some type $t_j$ that would be the only type to have a payoff strictly higher than her Nash equilibrium payoff if the responders were to believe she was type $t_j$.

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**Case (i): Signaler knows her cost parameter $s_i$ but not her value $v_j$**

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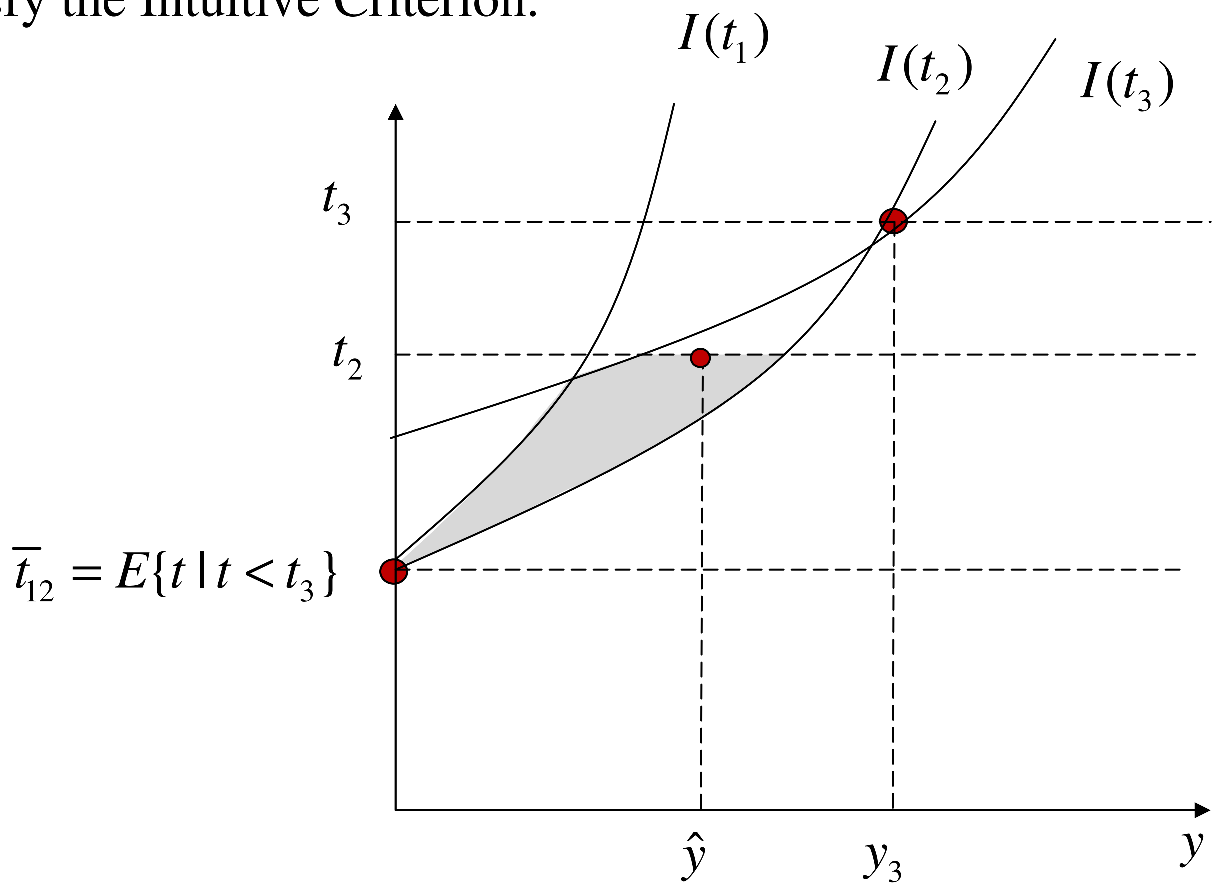
They ask whether there is some type $t_j$ that would be the only type to have a payoff strictly higher than her Nash equilibrium payoff if the responders were to believe she was type $t_j$.

In this case the following cheap talk by the signaler is credible. "I am type $t_j$ and you should believe me because, if you do, I will gain from your response but no other type would gain from such a response."

If this is the case, then the NE fails the Intuitive Criterion.

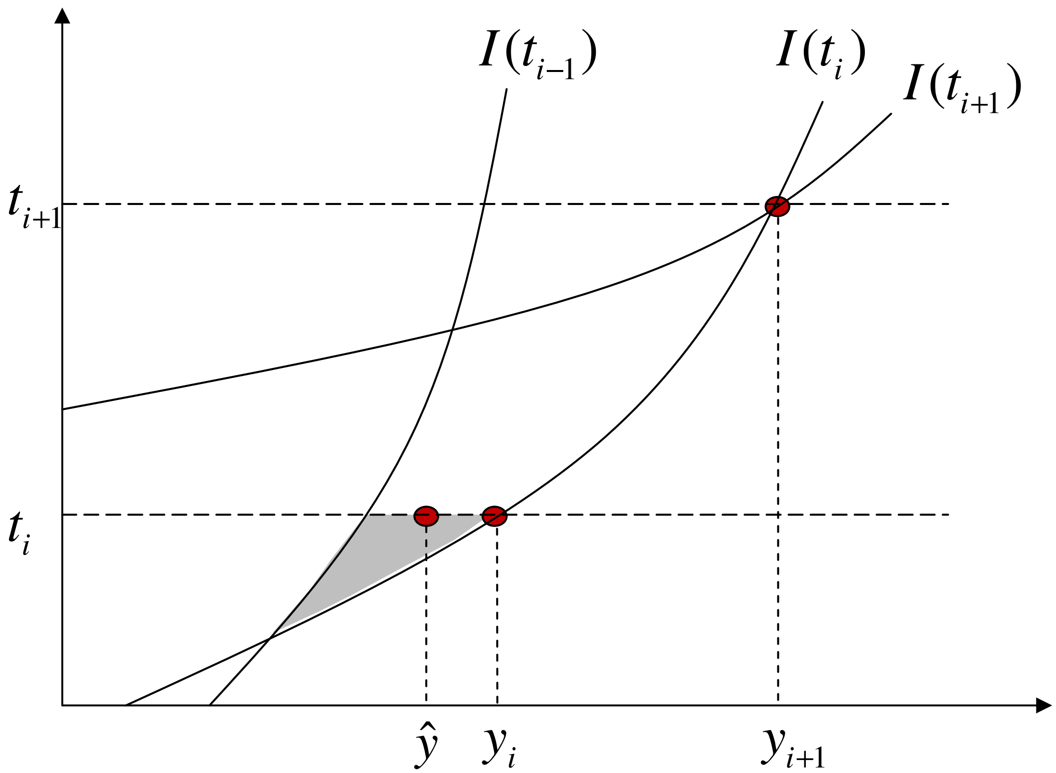
# REFINEMENTS - Intuitive Criterion

**Proposition**: If the signaler does not know his value, no NE with a pool can satisfy the Intuitive Criterion.



# REFINEMENTS - Intuitive Criterion

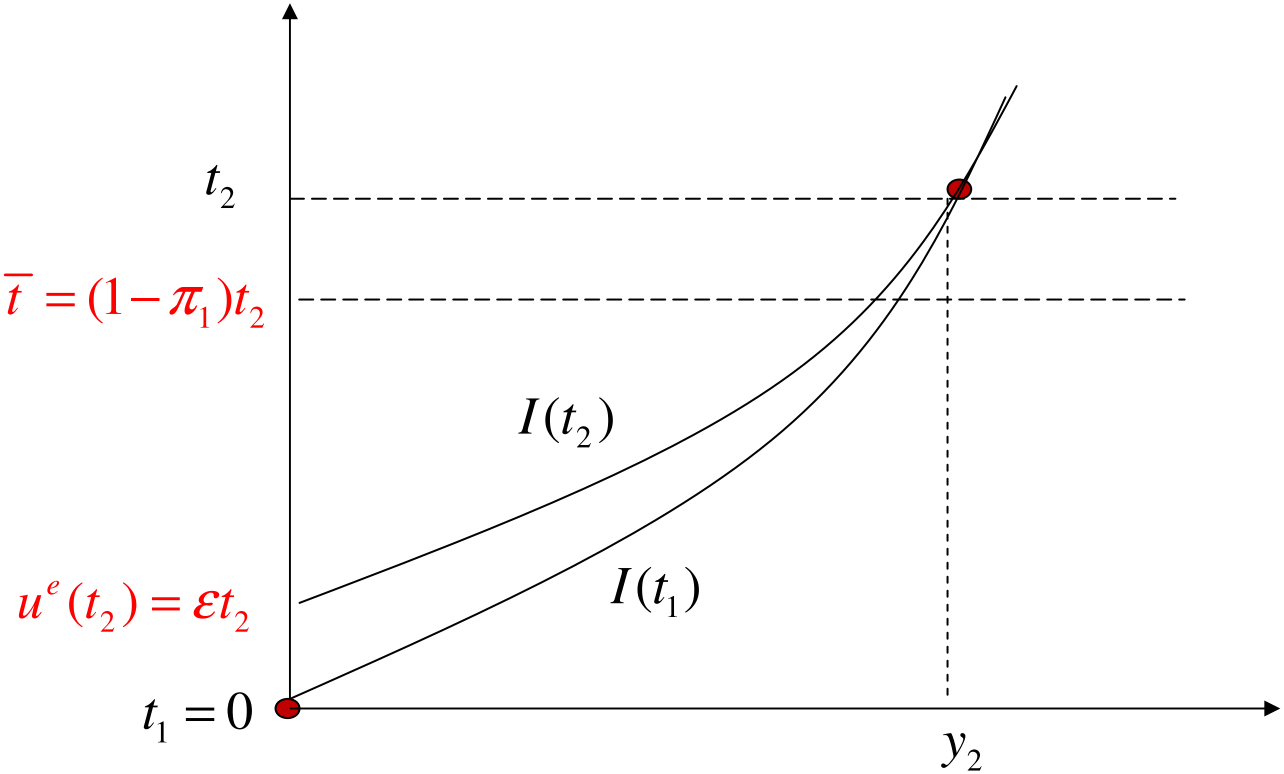
**Proposition**: If the signaler does not know his value, the NE must be "tight" in order to satisfy the Intuitive Criterion. (local upward constraints are binding.)



# REFINEMENTS - Intuitive Criterion

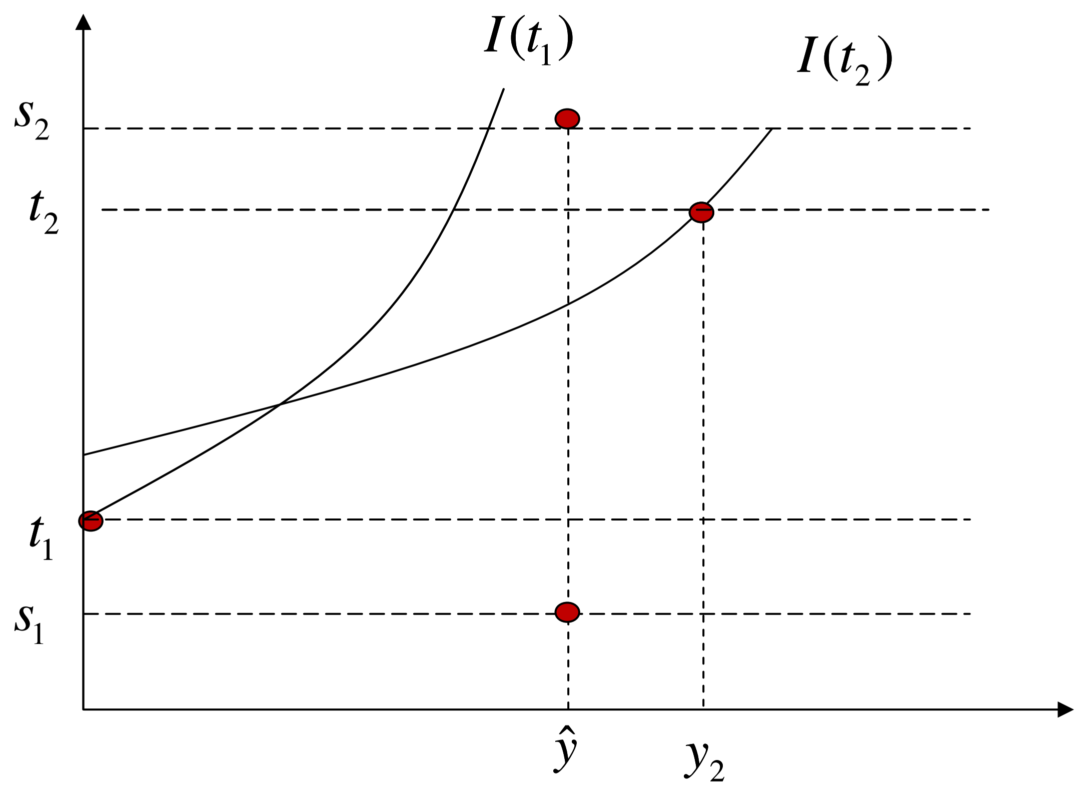
The separating NE can be a very undesirable outcome

Example: $t_1 = 0,\ C(t_2, y) = (1-\varepsilon)C(t_1, y)\quad \pi_1 = \Pr\{t = t_1\}$



# REFINEMENTS - Intuitive Criterion

**Proposition**: If the signaler knows his cost and value characteristics, non-tight Nash Equilibria and pooling equilibria satisfy the Intuitive Criterion.

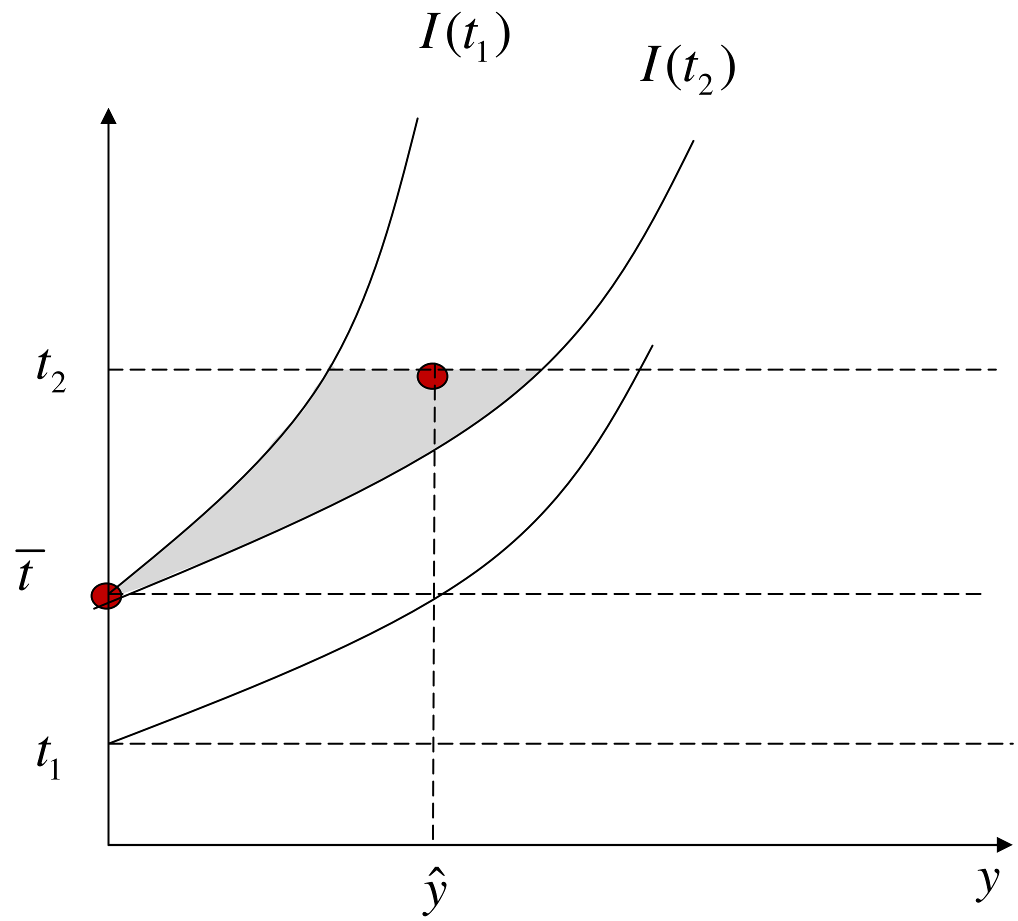



Suppose $(s_2, v_2)$ tries cheap talk…

Note that $t_i = E\{v \mid s = s_i\}$ lies between $s_1$ and $s_2$

# REFINEMENTS - Intuitive Criterion

Is the cheap talk really credible for the high value type in a pool?



# REFINEMENTS

Conclusion:

Standard refinements are very unsatisfactory

- Equilibrium outcome can be far worse than a pooling equilibrium for the good guys.
- Refinement does not work if the correlation between signaling cost parameter *s* and value parameter *v* is imperfect and the signaler has private information about both *s* and *v*.
- Big question about the credibility of the cheap talk for a pooled type

# Grossman – Perry Refinement

## Sequentially perfect Nash Equilibrium

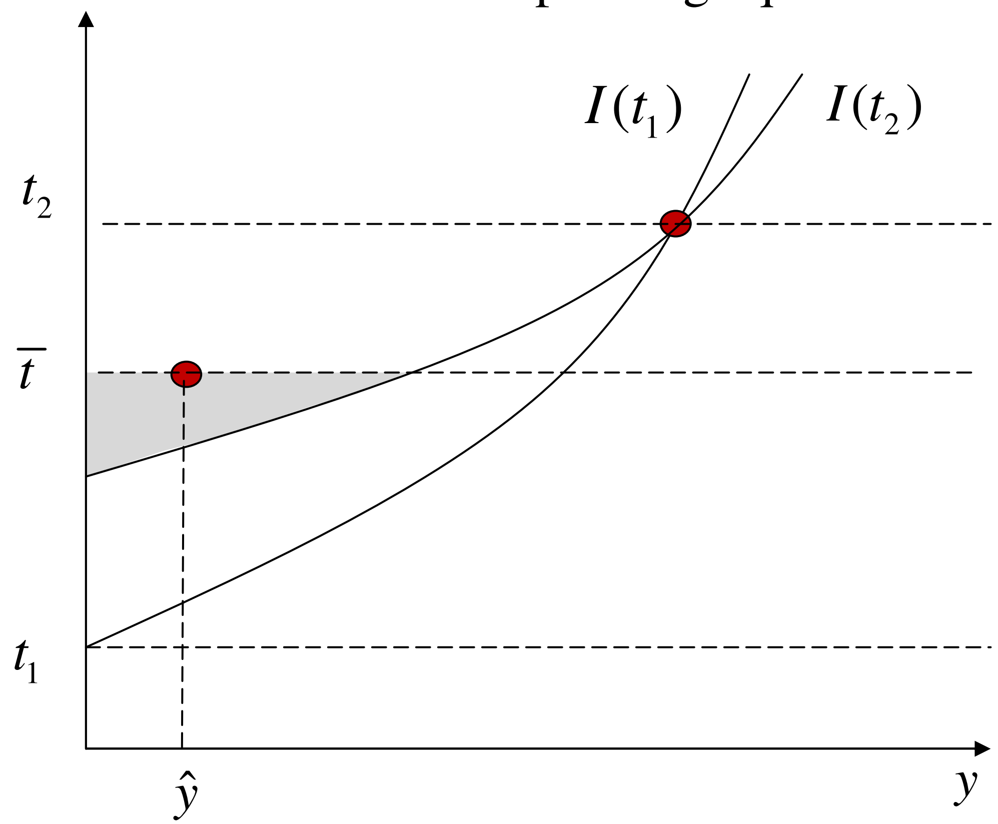
The responders observe the out-of-equilibrium signal $\hat{y}$.

They ask whether there is some subset of types $\hat{T}$ that would be the only types to have a payoff strictly higher than their Nash equilibrium payoff if the responders were to correctly predict she was in this subset (and hence update beliefs using Bayes Rule).

**Note: Grossman-Perry SPNE avoids critique (ii). Hence equilibrium cannot be pooling.**

REFINEMENTS Grossman –Perry **There may be no equilibrium.**

Consider the case in which the signaler knows her signaling cost but not her productivity. Since G-P is stronger than the Intuitive Criterion the only NE that can satisfy the G-P criterion is the separating equilibrium.



New refinement:

**Local Credibility Test**

Consider a NE such that $y(t_i) < y(t_{i+1})$. Define $T_i$ to be those types choosing $y(t_i)$ and $T_{i+1}$ to be the subset of types choosing $y(t_{i+1})$. Then consider local deviations to $\hat{y}$ between $y(t_i)$ and $y(t_{i+1})$, that is deviations by the following three subsets $\{T_i, T_{i+1}, T_i \cup T_{i+1}\}$. A Nash Equilibrium fails the LCT if for some $\hat{T} \in \{T_i, T_{i+1}, T_i \cup T_{i+1}\}$, a best response to $\hat{T}$ leaves these types (and only these types) better off.

LCT page 2

# Local Credibility Test

Proposition: A Nash Equilibrium that satisfies the LCT is “tight.” (The local upward constraints are binding.)

Proof: Follows that for the Intuitive Criterion

(IC page 5)

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Proposition: A Nash Equilibrium that satisfies the LCT is "tight." (The local upward constraints are binding.)

Proposition: The pooling equilibrium in which all types choose $y=0$ satisfies the LCT.

Proof: With a single pool at $y=0$ the only deviation is by the entire pool with higher $y$ so all are worse off.

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Proof: Suppose false and there is separating at the bottom. Then the local upward constraint is binding thus

$u(t_1,t_1,0)=u(t_1,t_2,y(t_2))$. Hence $y(t_2)\to 0$ as $t_2 \to t_1$.

Also

$$U^{SE}(t_2)-U^{SE}(t_1)=u(t_2,t_2,y(t_2)-u(t_1,t_2,y(t_2)).$$

Hence $\lim\limits_{t_2 \downarrow t_1}\dfrac{U^{SE}(t_2)-U^{SE}(t_1)}{t_2-t_1}=\lim\limits_{t_2 \downarrow t_1}\dfrac{u(t_2,t_2,y(t_2)-u(t_1,t_2,y(t_2))}{t_2-t_1}=u_1(t_1,t_1,0)$

Proposition: If types are sufficiently close and the LCT holds, there must be a pool of all types below some threshold

Proof: (continued)

With pooling,

$$U^P(t_2) - U^P(t_1) = u(t_2, a(t_2), 0) - u(t_1, t_1, 0)$$

Where $a(t) = E\{\tilde{t} \mid \tilde{t} \le t\}$

$$\lim_{t_2 \downarrow t_1} \frac{U^P(t_2) - U^P(t_1)}{t_2 - t_1} = u_1(t_1, t_1, 0) + u_2(t_1, t_1, 0)a'(t_1) > u_1(t_1, t_1, 0) \cdot$$

Thus both types are better off in the pool so an out-of-equilibrium action $\hat{y}$ sufficiently close to zero is credible and profitable.

Spence Model with multiplicative cost:

$$u(t,\hat{t},y)=\hat{t}-\frac{e(y)}{h(t_i)}$$

WOLOG we may assume this is linear in *y*.

$$u(t,\hat{t},y)=\hat{t}-\frac{y}{h(t_i)}$$

$$F(t_i)=\Pr\{\tilde{t}\le t_i\},\;\; f_i=F(t_{i+1})-F(t_i)$$

Proposition: If a NE is separating in the neighborhood of $t_i$ then, in the limit as the number of types grows large, it satisfies the LCT at $t_i$ if and only if

$$\frac{H''(t_i)}{H'(t_i)}\ge\frac{F''(t_i)}{F'(t_i)},$$

That is $H$ is more convex than $F$

LCT 7

# How does the proof work? (**Comment on two type case**)

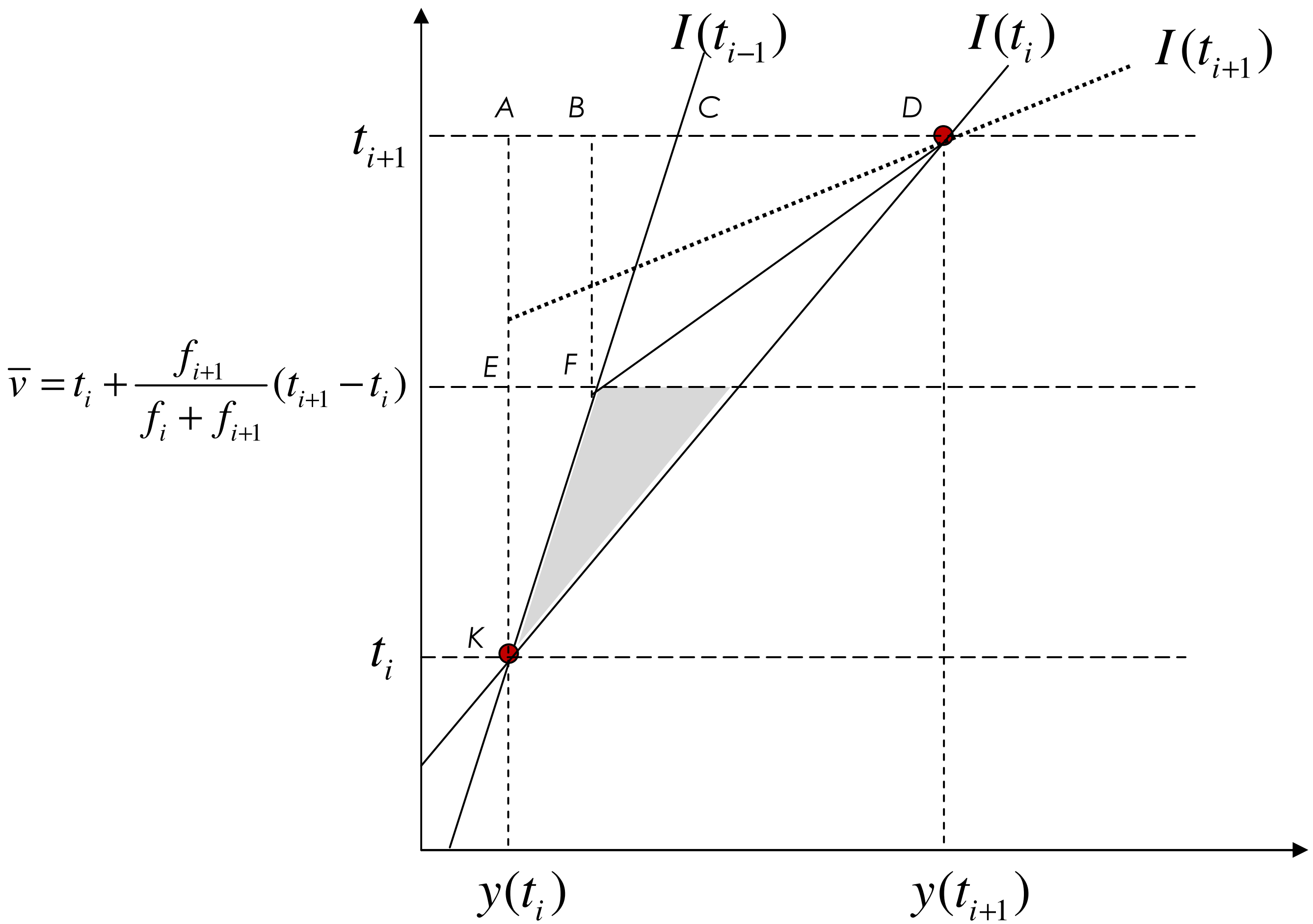


Fig. 3.3: Applying the LCT

**Proposition 4.6: Weak and strong signals (large number of types)**

Let $U_i(t),\ i=1,2$ be equilibrium payoffs when the cost of signaling is

$C_i(t,y) = y / H_i(y)$. Suppose that types signal if and only if $t \geq \tau$.

If $\dfrac{H_1'(t)}{H_1(t)} > \dfrac{H_2'(t)}{H_2(t)},\ t > \tau$ then $U_1(t) > U_2(t),\ t > \tau$.

LCT 9

**Proposition 4.7: Weak Signals and Pooling (large number of types)**

If $H'(t)/H(t) < F'(t)/F(t),\ t \in [0,1]$ then the unique Nash Equilibrium that satisfies the LCT is the pooling equilibrium.

LCT 10

**Proposition 4.9**: **Strong signals and signaling equilibria**

Suppose $H(t_0) > 0$, $H'(t_\infty)/H(t_\infty) > F'(t_\infty)$ and the necessary and sufficient condition for a separating equilibrium to satisfy the LCT holds for all $t \in [t_0, t_\infty]$. Then, there exists $\tau^*$ such that for any $\tau \geq \tau^*$ there exists a Nash Equilibrium satisfying the LCT with pooling of all types below $\tau$ and separation of types for all higher $t$. The higher is $\tau$ the greater the utility of the low types and the lower the utility of all those that separate.

## Strong Local Credibility Test

Consider a NE such that $y(t_i) < y(t_{i+1})$. Define $T_i$ to be those types choosing $y(t_i)$ and $T_{i+1}$ to be the subset of types choosing $y(t_{i+1})$. Then consider all local deviations to $\hat{y}$ between $y(t_i)$ and $y(t_{i+1})$, that is deviations by the types in any subset $\hat{T}$ of $T_i \cup T_{i+1}$. A Nash Equilibrium fails the strong LCT if for some $\hat{T}$, a best response to $\hat{T}$ leaves these types (and only these types) better off.

This is the local version of the Grossman-Perry Criterion.

Strong LCT 2

Proposition 5.1: A NE with pools cannot satisfy the Strong Local credibility Test

Strong LCT 3

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Proposition 5.2: If higher types have sufficiently higher reservation utility levels, there is a unique NE satisfying the strong LCT.

(Given the previous proposition this must be separating.)

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Focus has been on forcing a uniqueness result in simple models (where the negative correlation between signaling cost and signaling value is perfectly correlated.)

We have generalized the standard model to allow for imperfect correlation and shown that by staying within the spirit of the refinements literature we are able to characterize a family of equilibria.

If the signaling technology is weak, the unique equilibrium is the efficient pooling equilibria. If the signaling technology is sufficiently strong there exist equilibria with signaling by all those above some threshold.